The Effect of Spectral Height on Low-Frequency Motion Amplitudes

Simplified Treatment for Lightly Damped Linear Systems and Rectangular Spectra

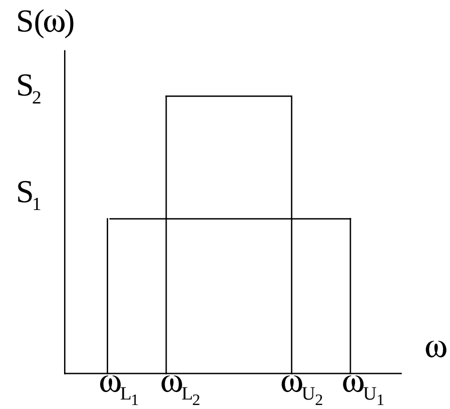
We examine the qualitative effect on low-frequency motions of a moored vessel to changes in wave spectral amplitude by analyzing vessel response to two rectangular wave spectra, S_1 and S_2, of equal variance. Waves associated with these spectra approach head-on to a vessel in a lightly damped linear mooring restraint. We assume for simplicity that regular-wave drift-force coefficients are frequency independent with constant value $f(\omega) = f_0$. The regular wave drift force F_{RW} is

$$F_{RW} = 0.5*f_0*D_w*B*\eta^2.$$

Here, D_w is the weight density of water, B is the beam of the vessel and η is wave amplitude. Under the stated conditions these spectra posses the same variance (σ_w^2) and associated mean wave drift force (F_{MD}); "U"pper and "L"ower rectangular frequency limits are ω_U, ω_L respectively:

$$\sigma_w^2 = S_1*(\omega_{U_1} - \omega_{L_1}) \equiv S_1*\Delta\omega_1 = S_2*(\omega_{U_2} - \omega_{L_2}) \equiv S_2*\Delta\omega_2,$$

$$F_{MD_1} = F_{MD_2} = f_0*D_w*B*\sigma_w^2.$$



The variable drift force spectrum at zero frequency is

$$S_F(0) \approx 2*(D_w*B)^2\int\{f(\omega)*S(\omega)\}^2 d\omega,$$

$$S_{F_1}(0) \approx 2*(D_w*B*f_0*S_1)^2*\Delta\omega_1 = 2*(D_w*B*f_0*\sigma_w^2)^2/\Delta\omega_1,$$

$$S_{F_2}(0) \approx 2*(D_w*B*f_0*S_2)^2*\Delta\omega_2 = 2*(D_w*B*f_0*\sigma_w^2)^2/\Delta\omega_2.$$

In particular, the ratio $S_{F1}(0)/S_{F2}(0)$ can be written simply

$$S_{F_1}(0)/S_{F_2}(0) = \Delta\omega_2/\Delta\omega_1 = S_1/S_2 .$$

Note that wave damping of these systems depends essentially on the *mean* drift force, which is the same for these two spectra so that *total* dimensionless system damping Γ, *including* wave-drift contribution, is the same for the two cases (damping in percent of critical $\equiv 50*\Gamma$).

The motion variance of a lightly damped linear spring-mass system with spring constant k, natural frequency ω_0 and dimensionless damping coefficient Γ is

$$\sigma_x^2 = E[x^2] = \pi*\omega_0*S_F(\omega_0)/(2\Gamma k^2).$$

We can as usual approximate $S_F(\omega_0) \approx S_F(0)$ so the ratio $\sigma_{x_1}/\sigma_{x_2}$ of rms motions becomes

$$\sigma_{x_1}/\sigma_{x_2} = \sqrt{\{S_{F_1}(0)/S_{F_2}(0)\}} = \sqrt{\{\Delta\omega_2/\Delta\omega_1\}} = \sqrt{\{S_1/S_2\}}.$$

Thus, all characteristic motions (rms, most probable peak, etc.) for a linear mooring system will increase roughly in proportion to the square root of the spectral height for constant spectral energy.

Note: Since Bretschneider and "Mean" JONSWAP spectra with the same energy and peak period differ in spectral height by about a factor of 2 for typically occurring peak periods, JONSWAP low-frequency vessel motions will be roughly 1.4 times larger than Bretschneider motions for a *linearly moored* vessel, other things being equal.